Cambridge
IGCSE

## Cambridge International Examinations

Cambridge International General Certificate of Secondary Education

## CANDIDATE

 NAMECENTRE NUMBER


## ADDITIONAL MATHEMATICS

Paper 2
February/March 2016
2 hours
Candidates answer on the Question Paper.
Additional Materials: Electronic calculator

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80 .

## Mathematical Formulae

## 1. ALGEBRA

## Quadratic Equation

For the equation $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Binomial Theorem

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n},
$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}$

## 2. TRIGONOMETRY

Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A \\
\Delta=\frac{1}{2} b c \sin A
\end{gathered}
$$

1 Two variables $x$ and $y$ are such that $y=\frac{5}{\sqrt{x-9}}$ for $x>9$.
(i) Find an expression for $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
(ii) Hence, find the approximate change in $y$ as $x$ increases from 13 to $13+h$, where $h$ is small.

2 The sets $A, B$ and $C$ are such that

$$
\begin{aligned}
& C \subset A, \\
& B \cap C=\varnothing, \\
& \mathrm{n}(A \cap B)=2, \\
& \mathrm{n}(B)=12, \\
& \mathrm{n}(B \cup C)=14, \\
& \mathrm{n}(A \cup B)=19 .
\end{aligned}
$$

Complete the Venn diagram to show the sets $A, B$ and $C$ and hence state $\mathrm{n}\left(A \cap B^{\prime} \cap C^{\prime}\right)$.


3 Find the equation of the curve which passes through the point $(1,7)$ and for which $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{9 x^{4}-3}{x^{2}}$.

4 (a) $\mathrm{f}(x)=a \cos b x+c$ has a period of $60^{\circ}$, an amplitude of 10 and is such that $\mathrm{f}(0)=14$. State the values of $a, b$ and $c$.
(b) Sketch the graph of $y=3 \sin 4 x-2$ for $0^{\circ} \leqslant x \leqslant 180^{\circ}$ on the axes below.


5 (i) Find, in ascending powers of $x$, the first 3 terms of the expansion of $(3+k x)^{7}$, where $k$ is a constant. Give each term in its simplest form.
(ii) Given that, in the expansion of $(3+k x)^{7}$, the coefficient of $x^{2}$ is twice the coefficient of $x$, find the value of $k$.

## 6 Do not use a calculator in this question.



The diagram shows two parallelograms that are similar. The base and height, in centimetres, of each parallelogram is shown. Given that $x$, the height of the smaller parallelogram, is $\frac{p+q \sqrt{3}}{6}$, find the value of each of the integers $p$ and $q$.

7 (a) Given that $\mathbf{A}=\left(\begin{array}{rrr}4 & 6 & 8 \\ -2 & 0 & 4\end{array}\right)$ and $\mathbf{B}=\left(\begin{array}{rrr}6 & 1 & 2 \\ 7 & -2 & 1\end{array}\right)$, find $\mathbf{A}-3 \mathbf{B}$.
(b) Given that $\mathbf{C}=\left(\begin{array}{rr}-2 & 0 \\ 4 & 1\end{array}\right)$ and $\mathbf{D}=\left(\begin{array}{rr}4 & 3 \\ -3 & -5\end{array}\right)$, find
(i) the inverse matrix $\mathbf{C}^{-1}$,
(ii) the matrix $\mathbf{X}$ such that $\mathbf{X D}^{-1}=\mathbf{C}$.

8 The line $2 y=x+2$ meets the curve $3 x^{2}+x y-y^{2}=12$ at the points $A$ and $B$.
(i) Find the coordinates of the points $A$ and $B$.
(ii) Given that the point $C$ has coordinates $(0,6)$, show that the triangle $A B C$ is right-angled.

$P Q R S$ is a quadrilateral with $P S$ parallel to $Q R$. The perimeter of $P Q R S$ is 3 m . The length of $P Q$ is 1 m and the length of $P S$ is $x \mathrm{~m}$. The point $T$ is on $Q R$ such that $S T$ is parallel to $P Q$. Angle $S R T$ is $\theta$ radians.
(i) Find an expression for $x$ in terms of $\theta$.
(ii) Show that the area, $A \mathrm{~m}^{2}$, of $P Q R S$ is given by $A=1-\frac{\operatorname{cosec} \theta}{2}$.
(iii) Hence find the exact value of $\theta$ when $A=\left(1-\frac{\sqrt{3}}{3}\right) \mathrm{m}^{2}$.

10 (a) The vectors $\mathbf{p}$ and $\mathbf{q}$ are such that $\mathbf{p}=11 \mathbf{i}-24 \mathbf{j}$ and $\mathbf{q}=2 \mathbf{i}+\alpha \mathbf{j}$.
(i) Find the value of each of the constants $\alpha$ and $\beta$ such that $\mathbf{p}+2 \mathbf{q}=(\alpha+\beta) \mathbf{i}-20 \mathbf{j}$.
(ii) Using the values of $\alpha$ and $\beta$ found in part (i), find the unit vector in the direction $\mathbf{p}+2 \mathbf{q}$. [2]
(b)


The points $A$ and $B$ have position vectors $\mathbf{a}$ and $\mathbf{b}$ with respect to an origin $O$. The point $C$ lies on $A B$ and is such that $A B: A C$ is $1: \lambda$. Find an expression for $\overrightarrow{O C}$ in terms of $\mathbf{a}, \mathbf{b}$ and $\lambda$.
(c) The points $S$ and $T$ have position vectors $\mathbf{s}$ and $\mathbf{t}$ with respect to an origin $O$. The points $O, S$ and $T$ do not lie in a straight line. Given that the vector $2 \mathbf{s}+\mu \mathbf{t}$ is parallel to the vector $(\mu+3) \mathbf{s}+9 \mathbf{t}$, where $\mu$ is a positive constant, find the value of $\mu$.

11 A curve has equation

$$
y=\frac{x}{x^{2}+1} .
$$

(i) Find the coordinates of the stationary points of the curve.
(ii) Show that $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=\frac{p x^{3}+q x}{\left(x^{2}+1\right)^{3}}$, where $p$ and $q$ are integers to be found, and determine the nature of the stationary points of the curve.

12 A particle $P$ is projected from the origin $O$ so that it moves in a straight line. At time $t$ seconds after projection, the velocity of the particle, $v \mathrm{~ms}^{-1}$, is given by

$$
v=9 t^{2}-63 t+90 .
$$

(i) Show that $P$ first comes to instantaneous rest when $t=2$.
(ii) Find the acceleration of $P$ when $t=3.5$.
(iii) Find an expression for the displacement of $P$ from $O$ at time $t$ seconds.
(iv) Find the distance travelled by $P$
(a) in the first 2 seconds,
(b) in the first 3 seconds.

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